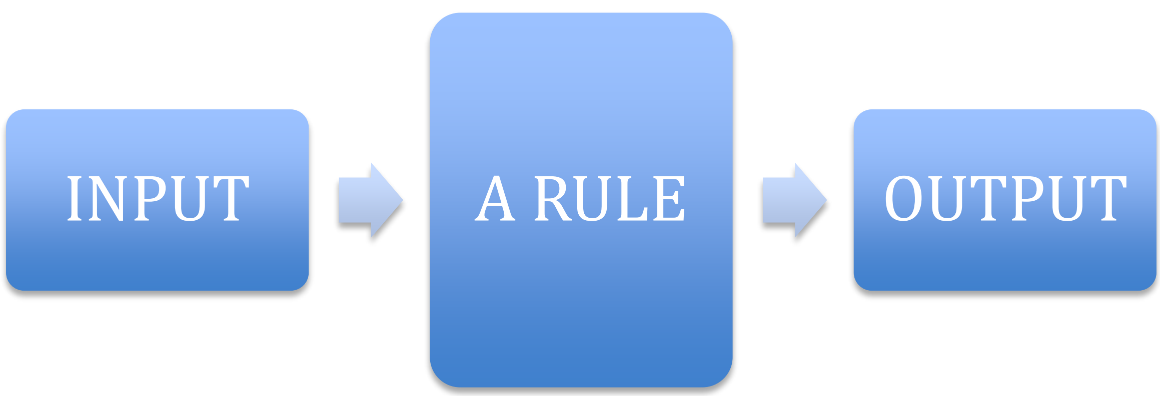
What is a Function?

Definitions:

- ***Relations* -** a collection (set) of ordered pairs. The first entry of an ordered pair is called the ***abscissa*** the second entry of the ordered pair is call the ***ordinate.***
- ***Domain* –** the set of abscissas of the relation.
- ***Range* –** the set of ordinates of the relation.
- ***Functions* –** a special type of relation in which a member of the domain is paired with **one and only one** member of the range.

A Function Machine

A useful analogy to describe a function is to think of it as a machine.



The **Input** of the function is the set of valid members of the domain. Once a member of the domain enters the machine a **rule** (often an Algebraic formula) controls what is to be done with the input. As an example, the rule might take the input and add three to it, as in $x + 3$, where x is the input. Once the rule has been applied to the input the machine sends (maps) the result to **output**. The output then becomes members of the range of the function. It is important to note that if the same input were re-entered into the machine, we would expect that the rule would deliver the same result (output) as before. If it does not, then it **is not** a function!

Consider the following relation:

$$f = \left\{(2,5), (-4,-1), \left(\frac{3}{4}, \frac{15}{4}\right)\right\}$$

The domain (or x coordinates) = $\left\{2, -4, \frac{3}{4}\right\}$ and the range (y coordinates) = $\left\{5, -1, \frac{15}{4}\right\}$. Thus the members of the domain are mapped to **one and only one** member of the range by the rule: $x + 3$.

Therefore, we have the following ***mapping*** of a function ***f***.

$f \mid 2 \rightarrow 5$
$f \mid -4 \rightarrow -1$
$f \mid \frac{3}{4} \rightarrow \frac{15}{4}$

Function Notations

In the above notation of the function ***f***, members of the domain are mapped (sent) to members of the range. As an example, in words, **the function *f maps (sends)* 2 to 5.** More generally we could state the function as:

$\boldsymbol{f \mid x \rightarrow x + 3}$ **-** The function *f* maps *x* to *x* + 3

Though the above function notation accurately describes a function, a more modern notation is often used by mathematicians, as shown below:

$\boldsymbol{f(x) = x + 3}$ - In words, ***f* of *x* is equal to *x* + 3.**

Restrictions on the Domains of Functions

In the function $\boldsymbol{f(x) = x + 3}$ the domain is all the **<u>valid</u>** *x* inputs of *f*. By valid we mean is there a value that *x* could take on that would not yield an output? For this example, *x* can be any **real** number because the function simply adds three units to whatever value of *x* we choose. So the domain can be expressed in *Set-Builder Notation as* $\{\boldsymbol{x}|\boldsymbol{x} \in \mathbb{R}\}$ or in *Interval Notation* as $(-\infty, \infty)$.

However, consider the function $\boldsymbol{g(x) = \frac{1}{x}}$ here *x* can take on any **real** number with the exception of zero! Why? Because if zero is the input for *g*, then $\boldsymbol{g(0) = \frac{1}{0}}$ which is undefined, by *definition of division by zero*! Therefore, the domain of *g* is $\{\boldsymbol{x} \in \mathbb{R}|\ \boldsymbol{x} \neq \boldsymbol{0}\}$ or $(-\infty, \boldsymbol{0}) \cup (\boldsymbol{0}, \infty)$**.**

In conclusion, restrictions on the domains are determined by the particular ***rule*** of the function.

Tests to Determine if a Relation is a Function

1. If the relation is expressed as ordered pairs, then examine all the abscissas (*x* coordinates) for repeats. If a repeat exists, then both output values **must** be the same. If the relation maps to a given input to more than one output, then the relation **is not** a function.

   $f\ |\ 5\ \rightarrow 8\ and\ f|\ 5\ \rightarrow\ -3$

2. **Vertical Line Test** – when a relation is graphed a vertical line is drawn through the graph. If the vertical line **only** intersects the graph **once** then the relation is a function. If the vertical line intersects the graph more than once then the relation **in not** a function.